

PROPAGATION OF STRESS WAVES IN THIN ORTHOTROPIC PLATES

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Abstract. The article deals with the issue of longitudinal and transverse wave propagation in the body of the thin viscoelastic orthotropic plate of Glass-Epoxy composite material. The plate has dimensions $0.1 \times 0.1 \times 0.0005$ m. This plate was circumferentially fixed and loaded by impact force 1 N in the centre of the plate (impact centre). The impact load was induced by the steel thin rod with spherical contact end. The radius of the rod c is 0.0025 m. For determination of the wave propagation of the plate the Kirchhoff model orthotropic plate, which is based on the Voight-Kelvin model, was used. The material properties of Glass-Epoxy are taken from literature material lists. These quantities are density, Poisson number, tensile modulus and shear modulus for x , y , and z directions. The calculation of displacements and velocities in the axes direction (x , y , z) was solved in MATLAB programme. The maximum calculation time reached 2×10^{-5} s, the time step of the calculation was applied up to 2×10^{-7} s. The waves are propagated from the impact centre in all directions. The wave propagation in x , y , z direction was determined at three points (B1, B2, B3 – position is presented in Figure 2) at a distance of 10 mm from the impact centre. The results, i.e. the displacements and the velocities at each observed point, are shown in Figures 3 to Figures 8. The coordinates of the points, in which the displacements and velocities were determined, are B1 [0.01; 0.00; 0.00], B2 [0.01; 0.01; 0.00] and B3 [0.00; 0.01; 0.00]. The results show that the velocity of the longitudinal wave propagation (i.e. in the x and y direction) is approximately 1.5 times greater than the velocity of a transverse wave dissemination (i.e. in the z direction). This result is in accordance with the wave theory.

Keywords: orthotropy, vibration, velocity, thin plate, wave propagation.

Introduction

Wave propagation problems investigated in the bodies, especially in plates and beams, have been in the scope of engineers and scientist since the end of the 18th century. At the beginning of the 19th century, Napoleon declared a financial reward to enhance the progress of this problem solution, which was one of the most national importances. This issue has not yet been satisfactorily resolved. A number of studies and monographs have been published [1-3], plate vibrations are a permanent topic, different tasks are dealt with [4], different effects are being analyzed, and different methods of solution are used [5-8]. The most sophisticated solution is the non-stationary oscillation of elastic isotropic plates [9]; the solution of orthotropic plates is limited to several studies [10-12]. The Faculty of Mechanical Engineering has been engaged in long-term research, dealing mainly with solutions of non-stationary character [13-15]. During that time, the Department of Machines and Mechanics established new approaches and solutions and worked on a comprehensive theory of wave propagation in thin plates.

There are a number of material and geometric models of solutions. The basic material model is the Hook's model, from geometric models it is the Kirchhoff's model, which defines a thin plate model considering only the vertical shifting of the board and its corresponding inertial effects. The Rayleigh's model arranges this basic model by incorporation of the effect of cross-section rotation and corresponding inertial effect. The Flügge's model considers the effect of shear on the resulting vertical displacement, the Timoshenko-Mindlin's model considers both influences - influence of the turning of the cross section and the effect of the shear.

Materials and Methods

In this article we deal with the propagation of shock waves in a thin plate for which the Kirchhoff's geometric model was chosen. The displacements and velocities of wave propagation are solved in the direction x , y and z . Calculation in MATLAB is based on simple assumptions that significantly affect the results

- *plate geometry* - the size and nature of its strain, layout, and the character of the excited load
- *rheological properties of the plate's material*

- *simplified assumptions within the framework of the used methods of solution* – the assumption of small deformations, the linearity of the relationship, and thus the superposition principle, neglect of the shear effect, etc.

For the computational model of the plate according to Kirchhoff's theory for the orthotropic plate, it is possible to derive the strain of the plate as a function of displacements in the direction of axes – x , y , z , resp. u , v , w . The solution is presented for a specific case, where the bar impacts perpendicularly to the thin elastic plate, based on the condition of equality of the contact point A shifts in both bodies in the direction z (Fig. 1).

$$w_{A1} = w_{A2} \tag{1}$$

$$w_{KA1} + w_{DA1} = w_{T2} - w_{KA2} - w_{DA2}$$

where w_{A1} – displacement of the touch pad A of the plate, m;
 w_{A2} – displacement of the touch pad A of the rod, m;
 w_{KA1} – oscillating displacement of the projection of point A to lower face in the z direction, m;
 w_{DA1} – displacement of the touch deformation of the upper face, m;
 w_{T2} – displacement of the falling rod as a rigid body, m;
 w_{K2} – displacement of the flexible loose rod, m;
 w_{D2} – displacement of the rod deformation, m.

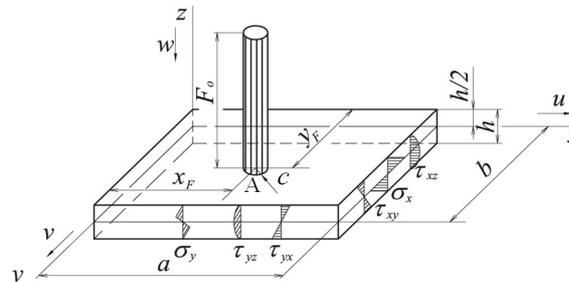


Fig. 1. **Plate model:** a, b, h – plate dimensions; c – impact radius; $F(t)$ – impact force; x_F, y_F – force coordinates; σ_x, σ_y – bending stresses; $\tau_{yx}, \tau_{xy}, \tau_{xz}, \tau_{yz}$ – shear stresses

Vertical displacement for the Kirchhoff's model of orthotropic thin plate $w(x, y, t)$

$$w(x, y, t) = \frac{16}{abc} \frac{F_0}{\rho h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{J_1(\gamma_{mn}c)}{\gamma_{mn}} \frac{\sin(\alpha_n x_F) \sin(\beta_m y_F)}{\omega_{mn}^2} \sin(\alpha_n x) \sin(\beta_m y) \sin^2\left(\frac{\omega_{mn}t}{2}\right)$$

$$\omega_{mn}^2 = \frac{1}{\rho h} (D_x \alpha_n^4 + 2D_{xy} \alpha_n^2 \beta_m^2 + D_y \beta_m^4) \tag{2}$$

$$\gamma_{mn} = \sqrt{\alpha_n^2 + \beta_m^2} \quad \alpha_n = n \frac{\pi}{a} \quad \beta_m = m \frac{\pi}{b}$$

$$D_x = \frac{E_x h^3}{12(1 - \mu_{yx}\mu_{xy})} \quad D_y = \frac{E_y h^3}{12(1 - \mu_{yx}\mu_{xy})} \quad D_{xy} = G_{xy} \frac{h^3}{12}$$

Angles of rotation tangent φ_x and φ_y (Fig. 2)

$$\varphi_x = \frac{\partial w}{\partial x} = \frac{16}{abc} \frac{F_0}{\rho h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{J_1(\gamma_{mn}c)}{\gamma_{mn}} \frac{\alpha_n}{\omega_{mn}^2} \sin(\alpha_n x_F) \sin(\beta_m y_F) \cos(\alpha_n x) \sin(\beta_m y) \cdot \sin^2\left(\frac{\omega_{mn}t}{2}\right) \tag{3a}$$

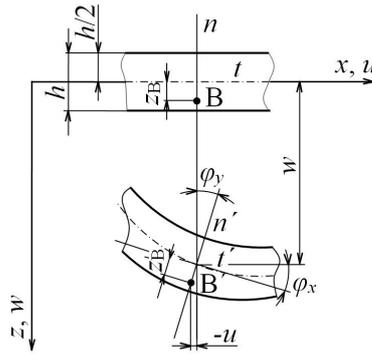


Fig. 2. **Rotation of plate element:** n, n' – normal; t, t' – tangents; u, w – displacement in x and y direction x and y ; φ_x, φ_y – angles of cross-sectional rotation in x and y direction; B, B' – control point; $z_B, z_{B'}$ – distance of point B ; h – thickness of plate

Angles of rotation tangent φ_x and φ_y (Fig. 2)

$$\varphi_y = \frac{\partial w}{\partial y} = \frac{16}{abc} \frac{F_0}{\rho h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{J_1(\gamma_{mn}c)}{\gamma_{mn}} \frac{\beta_m}{\omega_{mn}^2} \sin(\alpha_n x_F) \sin(\beta_m y_F) \sin(\alpha_n x) \cos(\beta_m y) \cdot \sin^2\left(\frac{\omega_{mn} t}{2}\right), \quad (3b)$$

Displacement u and v in x and y direction

$$u = -z\varphi_x \quad v = -z\varphi_y, \quad (4)$$

Velocity of wave $\dot{u}, \dot{v}, \dot{w}$

$$\dot{u} = \frac{\partial u}{\partial t} \quad \dot{v} = \frac{\partial v}{\partial t} \quad \dot{w} = \frac{\partial w}{\partial t}, \quad (5)$$

- where a, b, h – plate dimensions, m;
 c – impact radius, m;
 ρ – density of plate, $\text{kg} \cdot \text{m}^{-3}$;
 F_0 – impact force, N;
 $J_1(\gamma_{mn}c)$ – Bessel's function of the first kind, the first order for the $\gamma_{mn}c$ argument, -;
 α, β – constants, m^{-1} ;
 m, n – matrix elements, -;
 x_F, y_F – coordinate of the circular load, m;
 x, y, z – coordinates of the control point, m;
 ω_{mn} – own frequency, Hz;
 t – time, s;
 D_x, D_y, D_{yx} – plate stiffness in individual direction, $\text{N} \cdot \text{m}$;
 E, G – tensile and shear modulus, Pa;
 μ_x, μ_y, μ_{xy} – Poisson number in individual direction.

Results and Discussion

The relations for calculating the displacements and velocities were numerically solved in MATLAB. The viscoelastic thin orthotropic plate “Glass-Epoxy” was solved, the material properties are given in Table 1. The plate has got a dimension of $0.1 \times 0.1 \times 0.0005$ m (Fig. 3). The impact load was caused by rod with diameter $2c = 0.005$ m. The value of the impact load was $F_0 = 1$ N. The force position was in the centre of the plate. The plate was fixed around a perimeter. The magnitude of the displacements and velocities induced by the propagating wave in the thin plate was determined in points B1, B2, B3 – see Table 2.

In Fig. 4, 5 and 6 the graphs of displacement u, v, w for the individual control points (B1 to B3) are shown. In Fig. 7, 8 and 9 the graphs of velocity “ $\dot{u}, \dot{v}, \dot{w}$ ” for the individual control points (B1 to B3) are shown.

Table 1

Material properties of Glass-Epoxy

Name	Des.	Unit	Value
Tensile modulus	E_x	Pa	3.86×10^{10}
	E_y	Pa	8.27×10^9
	E_z	Pa	8.27×10^9
Modulus of shear elasticity	G_{yz}	Pa	3.45×10^9
	G_{xz}	Pa	4.14×10^9
	G_{xy}	Pa	4.14×10^9
Poisson's number	μ_{yz}	-	0.2
	μ_{zx}	-	0.043
	μ_{xy}	-	0.2
Density	ρ	$\text{kg}\cdot\text{m}^{-3}$	1800

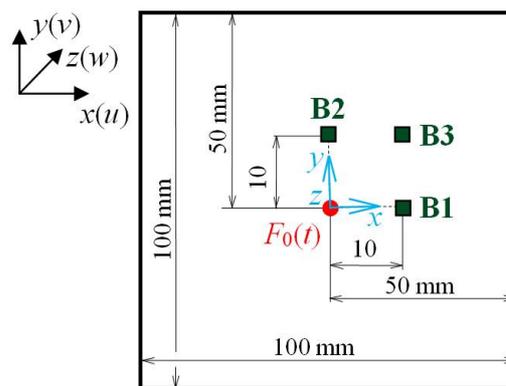


Fig. 3. Plate dimensions and impact center: $F_0(t)$ – impact load, B1, B2, B3 – control points

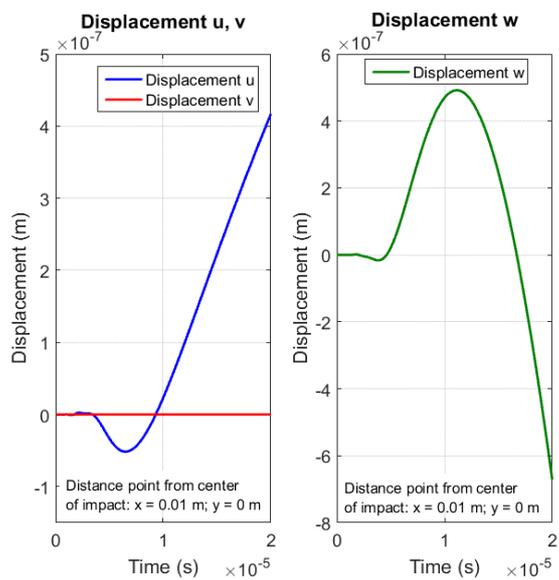


Fig. 4. Displacement “u, v, w” in point B1

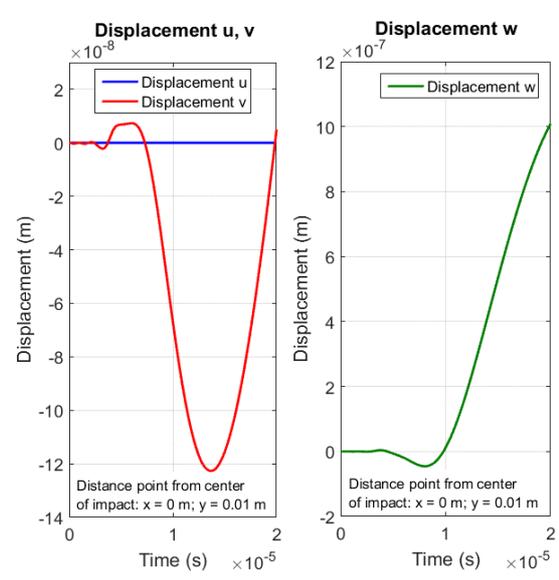


Fig. 5. Displacement “u, v, w” in point B2

By comparing the variations in Fig. 4 with Fig. 6, the variations are different for B1 and B3. This is due to the fact that the material properties are different in the direction of the x and y axes – it is actually an orthotropic material.

The displacement at point B3 shows a relatively equal course in the x and y axis. The amplitude is different, which means that it is an orthotropic material. Point B3 is away from the centre of impact of

0.0141 m (points B1 and B2 – 0.01 m). Fig. 4, 5 and 6 show different amplitude (B3 in both directions smaller). This is due to material attenuation (distance is greater than 0.0041 m).

Table 2

Control points

Des. points	Position of points from the centre of impact	
	x, m	y, m
B1	0.01	0.00
B2	0.00	0.01
B3	0.01	0.01

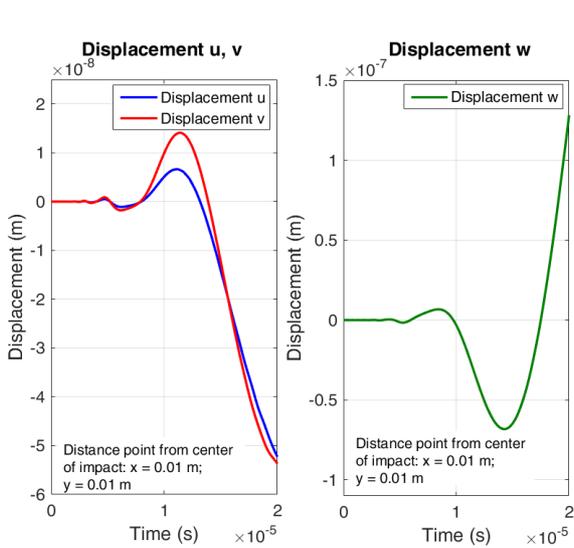


Fig. 6. Displacement “u, v, w” in point B3

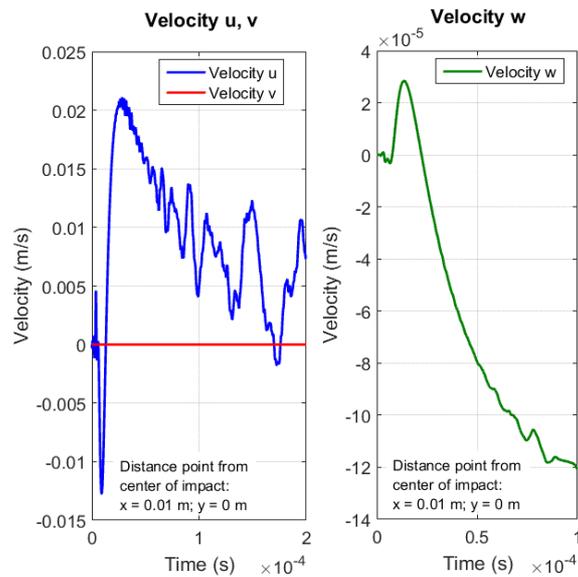


Fig. 7. Velocity “u,v,w” in point B1

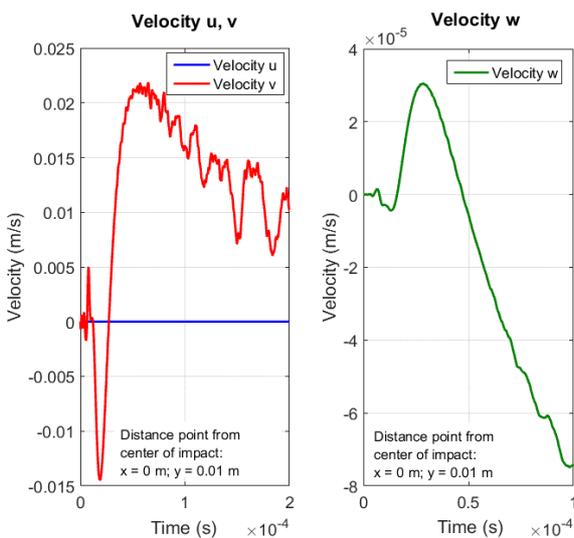


Fig. 8. Velocity “u,v,w” in point B2

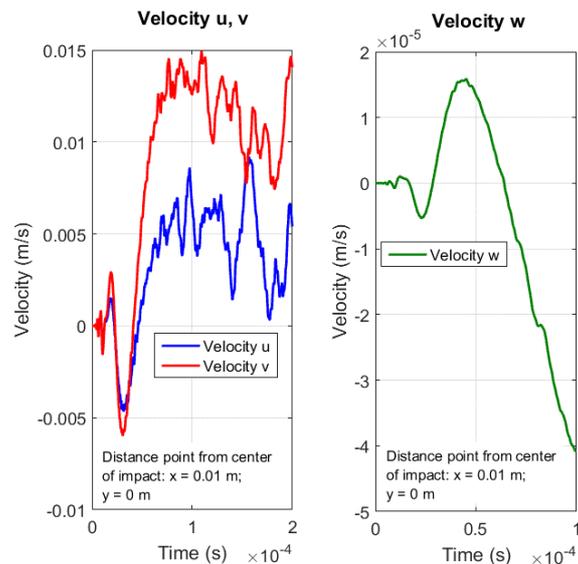


Fig. 9. Velocity “u,v,w” in point B3

The amplitudes in the direction x and y (points B1, B2) are different, as well as the times, see Fig. 6, 7 and 8. This is due to material attenuation in the axes - the orthotropic material. At point B3, there

is a clear displacement of the peaks at the x -axis versus y -axis (different material properties), as well as the amplitudes are different.

Conclusions

The oscillation of the orthotropic plate, which had different rheological properties in the x and y axes, was solved. Z -axis properties are the same at all control points. The difference in these properties ($E_x \neq E_y$, $G_{yz} \neq G_{xz}$) also manifests itself at different velocities in these axes. The course of deflection in x and y corresponds well to rheological properties. This difference is reflected in the velocity of the wave propagation (time t) and the amplitude of both displacement and velocity.

The further step of the investigation is to perform the Timoshenko-Mindlin calculation and compare it with the experiment.

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